Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Assessment of the Accuracy of Representing a Helical Vortex by Straight Segments"

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Nomenclature

p	=	reduced helix pitch
R	=	helix radius
U_b	=	binormal component of induced velocity
U_t	=	tangential component of induced velocity
β	=	inclination of vortex filament
Γ	=	circulation in vortex
θ	=	angular displacement along vortex

REFERENCE 1 presents a very useful assessment of the accuracy of representing a helical vortex by a sequence of straight segments, with specific reference to the errors associated with the helices of small pitch found in the wakes of helicopters, wind turbines, and propellers. An additional application, which the authors might not be aware of, is to the leading-edge vortex breakdown phenomenon encountered over delta wings at high angles of attack, which in many (if not all) cases also has a helical structure of small pitch.[2,3]

The authors consider three test cases for the velocity induced by a semi-infinite helical vortex, all three having control points in the plane of the start of the vortex. The control point for case 1 is on the centerline, for case 2 it shares the helix radius but is displaced by 180 deg from the helix start, and for case 3 it is at the start of the helix. For the first case the induced velocity is directed along the x axis and has an exact solution $U = p^{-1}$. For the latter two cases only the binormal components U_b of the induced velocities are considered, for which, apparently, no analytical solutions exist.

However, it might be of some relevance to note that for case 3 there is an analytical solution for the tangential component U_t of the self-induced velocity. This component is generally neglected in theoretical analyses of infinite helical vortices because it does not affect the apparent motion of the helix as a whole. Nevertheless, there are three aspects of "aeronautical" (i.e., semi-infinite) helical vortex flows where the tangential component might be of some importance: first, in its contribution to the axial velocity distribution within the vortex core; second, when the vortex is generated by a moving source such as a rotor tip; third, when a straight vortex "breaks down" into a spiral burst. In the first instance changes in the axial velocity profile will affect the stability of the vortex core, whereas in the latter two instances the axial component of the tangential velocity contributes to the overall convection velocity and, hence, the pitch of the vortex as it is generated.

Using the notation of Ref. 1 (where velocities are nondimensionalized by $\Gamma/4\pi R$ and lengths by R), the Biot–Savart law for the tangential component of the self-induced velocity at the start of a semi-infinite vortex can be written[4] as

$$U_t = \frac{p}{\sqrt{1+p^2}} \int_0^\infty \frac{2(1-\cos\theta) - \theta\sin\theta}{[2(1-\cos\theta) + p^2\theta^2]^{\frac{3}{2}}}\, d\theta \qquad (1)$$

where θ is an angular parameter defining position along the helix and p is the reduced helix pitch. In the interest of avoiding confusion, it is probably also worth noting at this point that the corresponding "physical" pitch of the helix (i.e., in the engineering sense) is $2\pi p$. Making the substitutions $\gamma = \theta/2$ and $x = (\sin\gamma)/\gamma$ gives the elementary integral

$$U_t = \frac{p}{\sqrt{1+p^2}} \int_0^1 \frac{x}{(x^2+p^2)^{\frac{3}{2}}}\, dx$$

and hence for a semi-infinite vortex

$$U_t = \frac{1}{\sqrt{1+p^2}} - \frac{p}{1+p^2} \qquad (2)$$

Noting that the helix inclination angle β is given by the relation $\tan\beta = p^{-1}$, this relation can also be written as

$$U_t = \sin\beta(1-\cos\beta) \qquad (3)$$

This rather simple result appears not to have been previously reported.

References

[1] Wood, D. H., and Li, D., "Assessment of the Accuracy of Representing a Helical Vortex by Straight Segments," *AIAA Journal*, Vol. 40, No. 4, 2002, pp. 647–651.

[2] Jumper, E. J., Nelson, R. C., and Cheung, K., "A Simple Criterion for Vortex Breakdown," AIAA Paper 93-0866, Jan. 1993.

[3] Greenwell, D. I., "Pitfalls in the Interpretation of Delta Wing Vortex Flow Visualisation Images," NATO RTO, Paper A-5, May 2001.

[4] Ricca, R. L., "The Effect of Torsion on the Motion of a Helical Vortex Filament," *Journal of Fluid Mechanics*, Vol. 273, 1994, pp. 241–259.

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